ON THE PROBABLE INTERPRETATION OF ANTICORRELATION BETWEEN THE PROTON TEMPERATURE AND DENSITY IN THE SOLAR WIND

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The anticorrelated distributions of temperature and density of protons are a well-known property of the solar wind. Nevertheless, it is unclear till now if they are formed by some kind of the universal physical mechanism? Unfortunately, a straightforward comparison of the characteristic relaxation times for the temperature and density, on the one hand, and pressure, on the other hand, encounters the problem of inapplicability of the hydrodynamical approach in the situation when the free-path length of the protons is considerably greater than the spatial scale of the structures under consideration. To resolve this problem, some kinds of the MHD turbulence—reducing the effective free paths—are usually assumed. In the present paper, we use an alternative approach based on the electrostatic (Langmuir) turbulence, described by the mathematical formalism of the spin-type Hamiltonians, which was actively discussed in the recent time in the literature on statistical physics. As follows from the corresponding calculations, formation of the anticorrelated distributions of temperature and density is a universal property of the strongly-nonequilibrium plasmas governed by the spin-type Hamiltonians when they gradually approach the thermodynamic equilibrium. So, just this phenomenon could be responsible for the anticorrelations observed in the solar wind.

Keywords: solar wind, strongly-nonequilibrium plasmas, relaxation of plasma irregularities.

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Introduction

It is well known that the solar-wind plasma flux, measured by spacecraft, experiences considerable fluctuations of all basic parameters, such as the density, temperature, chemical composition, *etc.* (The general reviews can be found, *e.g.*, in [1, 2] and references therein.) On the other hand, these fluctuations exhibit the persistent correlations with each other, which were analysed in detail in a number of previous works [3, 4].

In the present paper, we pay attention only to one kind of such correlations, namely, between the temperature of protons (which are the major fraction of positive ions in the solar wind) and their density. The corresponding observational data are illustrated in Fig. 1, which is a particular example of the anticorrelated distributions of T and n. It was drawn by the one-minute values of the respective parameters from the public database OMNIWeb: `https://omniweb.gsfc.nasa.gov`. In general, the correlation diagram exhibits a hyperbolic shape, which corresponds to the negative correlation coefficient. Here, its particular value is $r = -0.13$ for the entire period from January 2009 to December 2017, *i.e.*, approximately during a single cycle of the solar activity. A quite similar value, $r = -0.15$, was found in the earlier work [3] (see Table 2 in that paper). Let us emphasize that the above-cited quantities represent the mean values for the very long time intervals. The absolute values of the correlation coefficient at the sufficiently short intervals (about a few hours) can be much greater; *e.g.*, Fig. 2 in paper [5], where $r = -0.68$.

One can see in Fig. 1 the well-expressed horizontal and vertical tails (parallel to the axes n and T, respectively) in the distribution of the observational points. This means that the greater values of density and temperature are anticorrelated with each other. Let us mention that we used here the linear rather than logarithmic scales, which were employed in the most of the previous works. The above-mentioned tails are well visible just in the linear scale, while they become compressed into a single spot in the logarithmic scale. Let us emphasize also that such correlation diagrams can be found also in a number of other publications by various authors. The aim of our figure is to demonstrate that the peculiarities found in the earlier papers survive in the last solar cycle (although a lot of parameters of the solar wind, in general, changed considerably during the last two cycles [6]).

Unfortunately, the most of theoretical works on cosmic plasmas at the present time are based on the numerical simulations. They can describe in detail some particular situations but are often unable to reveal the general physical laws governing the corresponding phenomena. This situation refers particularly to the interpretation of the above-mentioned anticorrelations of temperature and density in the solar wind. Yet another problem is that the theoretical models of the solar wind are commonly formulated under assumption of incompressibility, just because the theory of MHD turbulence is developed in detail for the incompressible medium. Next, the fluctuations of the density are taken into account by the perturbation theory, which does not always allow their unambiguous and self-consistent treatment. As an example of such an approach, see paper [5] and references therein.

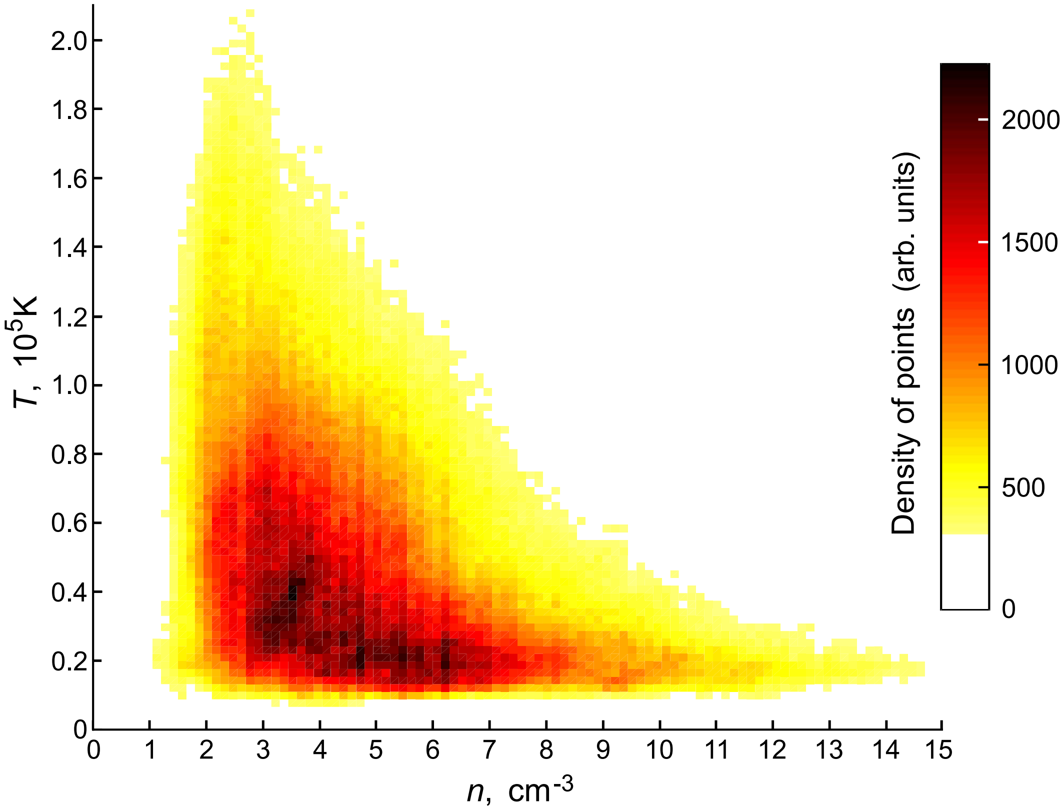

Figure 1: Correlation diagram of the temperature and density of the solar wind near the Earth in the period from January 2009 to December 2017. The density of points is the number of measured values within a unitary cell of size $(0.15\,\mathrm{cm}^{-3})\times(2200\,\mathrm{K})$. For better visualization, the values below 300 points per cell were cut off

1. Anticorrelations in the Simple Gas-dynamic Model

At the first sight, a rather general interpretation of the anticorrelations can be derived from the straightforward gas-dynamic analysis of the characteristic relaxation times for various physical quantities:

1. Equilibration of the irregularities in the strongly-nonequilibrium plasmas should involve both the "fast" and "slow" stages, as follows from the two very different time scales for the relaxation of fluctuations in pressure, $(\Delta t)_p$, on the one hand, and in the density and temperature, $(\Delta t)_n$ and $(\Delta t)_T$, on the other hand. Indeed, the characteristic rate of the relaxation in pressure is determined by the speed of sound,[1] $c_s \sim \lambda/\tau$ (where λ is the free-path length, and τ is the time of interparticle collisions). Then, the characteristic time for the relaxation of pressure in the domain of size Δl can be estimated as
$$(\Delta t)_p \approx \Delta l / c_s \sim (\Delta l / \lambda)\, \tau. \tag{1}$$

[1] The term "relaxation" does not necessarily imply here a damping of the oscillations of pressure. Instead, this could be just an establishment of some average value after a few oscillations.

On the other hand, the evolution of density and temperature is described by the equations of parabolic type: $\partial n/\partial t = -\mathrm{div}(k_n \nabla n)$ and $\partial T/\partial t = -\mathrm{div}(k_T \nabla T)$, where $k_n \approx k_T \sim \lambda^2/\tau$. Consequently, the corresponding relaxation times are estimated as

$$(\Delta t)_{n,T} \approx (\Delta l)^2 / k_{n,T} \sim (\Delta l/\lambda)^2 \tau. \qquad (2)$$

Since $\Delta l/\lambda \gg 1$, then $(\Delta t)_{n,T} \gg (\Delta t)_p$, *i.e.*, the equilibrium value of pressure throughout the entire system should be established very quickly, while the fluctuations of density and temperature relax at the much longer time interval.

2. In the conditions when a mean pressure was established, it follows immediately from the equation of state for an ideal gas, $p = nkT$ (where k is the Boltzmann constant), that the long-lived fluctuations of density Δn and temperature ΔT will be anticorrelated with each other: the smaller is the density, the greater should be the temperature, and *visa versa*.

Unfortunately, despite of universality of the above arguments, their application to the case of the solar wind encounters the problem of applicability of the hydrodynamic approximation. Really, if we take the typical proton temperature $T \approx 10^5\,\mathrm{K}$ and their typical density $n \approx 10\,\mathrm{cm}^{-3}$ [2] and then use the well-known expressions for scattering of charged particles by each other (*e.g.*, formulas (3.14) and (3.15) in [7]), we get the free-path length[2] $\lambda \approx 10^9\,\mathrm{km}$. This value is very large even in comparison with the total distance from the Sun to the Earth, $1.5 \cdot 10^8\,\mathrm{km}$. On the other hand, the above-discussed anticorrelations correspond to the characteristic scales of irregularities that are less by a few orders of magnitude. Indeed, since the typical speed of the solar wind passing through the measuring apparatus is about 500 km/s, then the fluctuations at the time scale of a few minutes correspond to the spatial fluctuations of size about a few tens or hundreds of thousands of kilometers. Therefore, to justify the applicability of the hydrodynamic approximation, the commonly-used models of the solar wind (see, for example, [8] and references therein) assume a presence of some kind of the MHD turbulence, considerably increasing the effective collisional frequencies.

It is the aim of the present paper to demonstrate that yet another approach to solving the above-mentioned problem might be the introduction of the electrostatic (Langmuir) rather than magnetohydrodynamic turbulence. A convenient mathematical formalism for the description of such turbulence are models of the nonequilibrium plasmas based on the spin-type Hamiltonians, which are actively developed in the last years.

Finally, let us mention that the anticorrelations of temperature and density in the framework of hydrodynamic equations were predicted already in the very old paper [5]; see equation (10) from that article. Unfortunately, this result depended crucially of the employed version of the perturbation theory: one variant of perturbations (which was called by the authors “Type I”) really resulted in the anticorrelations, while the second variant (“Type II”) led to the positive correlations. So, it remained unclear which of these cases is realized in the particular physical situations.

[2] We imply here the scattering of protons by protons, while the electrons represent a uniform neutralizing background.

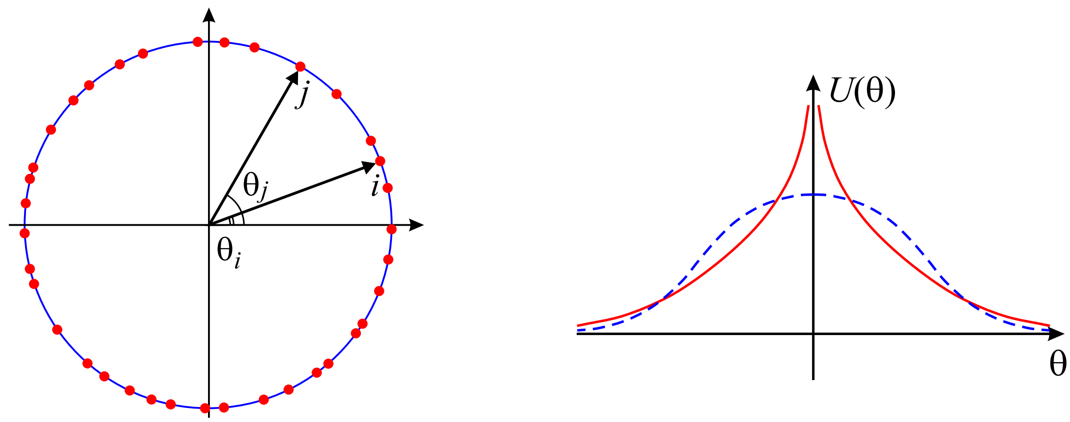

Figure 2: Description of plasmas by the spin-type models: (left-hand panel) geometric configuration of the one-dimensional system, and (right-hand panel) approximation of the Coulomb potential of interparticle interaction (solid red curve) by the Fourier harmonic of the lowest order (dashed blue curve)

2. Anticorrelations in the Strongly-Nonequilibrium Plasmas Described by the Spin-Type Hamiltonians

The method of description of the long-range (particularly, Coulomb) many-body systems by the spin-type Hamiltonians originated in the literature on statistical physics about 10 years ago (see, for example, [9, 10] and references therein) but remains poorly known in the plasma-physics community till now. Therefore, it is reasonable to resume here its basic ideas.

As is known, the Hamiltonian theory of the spin systems is a very old and mathematically well-developed branch of theoretical physics, whose history lasts for almost a century [11, 12]. Its original and most important scope of applicability was the solid-state physics and, first of all, the theory of magnetic phenomena. However, the spin-type Hamiltonians were used subsequently also for modelling a lot of other physical phenomena, *e.g.*, the strongly-nonequilibrium phase transformations in various theories with the spontaneous symmetry breaking [13, 14], and even in the adjacent scientific disciplines, such as biophysics.

As regards plasmas and the ensembles of particles with gravitational (Newtonian) interaction, the spin-type Hamiltonians are introduced there in the following way. Let us consider a one-dimensional system of ions, which is a reasonable approximation of the strongly-magnetized plasmas. Since the plasma as a whole should be electrically neutral, the set of ions is assumed to be immersed into a uniform electronic gas, which corresponds to the well-known approximation of the "one-component plasma" (OCP) [15]. Next, for the sake of simplicity we shall assume the periodic boundary conditions; so that this system will be topologically equivalent to the ring. Then, from the mathematical point of view, positions of the ions can be formally characterized by the angles θ_i, as shown in the left panel of Fig. 2.

Next, the Coulomb potential $U(\theta)$ can be approximated in the simplest case by a single Fourier harmonic of the lowest order, *i.e.*, the cosine function with period corresponding to the size of the entire system, as shown in the right panel of the same figure. Strictly speaking, the Fourier expansion of the Coulomb potential is ill defined because of its singularity in zero. So, the approximation by only one (the first) harmonic is a quite crude approximation of the general shape of potential. Nevertheless, since the first harmonic "globally" covers the entire system, it can reasonably represent the long-range character of the Coulomb (or gravitational) interactions.

Finally, Hamiltonian of the ions can be written in the form similar to the so-called Heisenberg Hamiltonian for a spin system:

$$\mathcal{H} = \frac{1}{2}\sum_{i=1}^{N} p_i^2 + \frac{J}{N}\sum_{i=1}^{N}\sum_{j=1}^{i-1}\left[1 - \cos(\theta_i - \theta_j)\right], \tag{3}$$

where p_i are the momenta of ions (which are assumed for simplicity to be of the unitary masses), N is their total number, and J is the coupling parameter commonly used in the spin models, which can be expressed through the parameters of the Coulomb or gravitational system. Let us mention that in the case of plasmas, where particles of the same charge are repelled from each other, the coupling parameter J in formula (3) should be negative. Following the terminology used in the solid-state physics, this corresponds to the "antiferromagnetic" type of interactions. On the other hand, at positive J ("ferromagnetic" interaction) the particles are attracted to each other; this corresponds to the gravitating systems.

In summary, the interaction between spins by the cosine law is analogous to the Coulomb interaction between the charged particles; the angle between two spins plying the role of distance between the particles in plasma. (The additional constant term in this formula was introduced just to get an exact equivalence with the spin system; it evidently does not affect the equations of motion.) Let us emphasize that the interaction between the ions is "collective": it is irreducible to a sequence of pair collisions. Besides, the Hamiltonian (3) evidently takes into account only electrical, not magnetic forces. Therefore, it describes the processes similar to the electrostatic (Langmuir) waves in plasmas.

As is known, the spin systems are characterized by the universal thermodynamic properties; and their equilibrium parameters can be usually calculated analytically, at least, in the one- and two-dimensional cases [11, 12]. On the other hand, the nonequilibrium properties depend on the initial and/or boundary conditions; and they are usually derived by combining the analytical and numerical methods. Particularly, dynamics of the strongly-nonequilibrium plasmas described by the Hamiltonian (3) was studied in detail in papers [9, 10]. Examples of these calculations are shown in Fig. 3. The left panel refers to the case of "antiferromagnetic" (Coulomb) interaction, when an external constant field was present, and the initial velocity distribution was non-Maxwellian. The right panel refers to the case of "ferromagnetic" (gravitational) interaction, with the Maxwellian initial velocity distribution perturbed by a short pulse of the external field.

Therefore, it is seen from these plots that a generic property of the relaxation of the nonequilibrium many-body systems described by the spin-type Hamiltonians is a formation of the anticorrelated spatial distributions of temperature and density at some intermediate stage of their evolution, before approaching the complete thermodynamic equilibrium. The

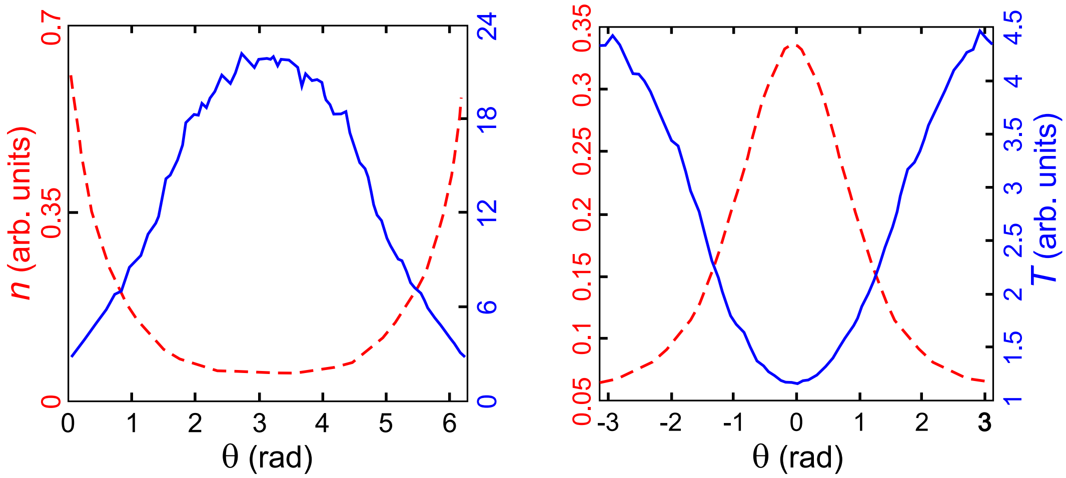

Figure 3: Examples of the anticorrelated distributions of temperature (solid blue curves) and density (dashed red curves), computed for the systems described by the spin-type Hamiltonians: (left-hand paned) Fig. 2 from paper [9], reprinted with permission from Springer Nature Switzerland AG, © 2014; and (right-hand panel) Fig. 2 from paper [10], reprinted with permission from American Physical Society, © 2015. The results are plotted in both panels in the dimensionless units used in the original works

entire such process can be illustrated by the diagram shown in Fig. 4:

1. It is assumed that the plasma was initially created in the state with considerable fluctuations of both the temperature T and density n that were independent of each other (left panel).

2. Next, this plasma quickly evolves to the quasi-stationary state where large fluctuations ΔT and Δn still survive but become anticorrelated with each other (middle panel).

3. Finally, at the much longer time interval, these fluctuations gradually decay, and the complete thermodynamic equilibrium is established (right panel).

It is important to emphasize that the characteristic time scales of the first and second stages of the relaxation, τ_1 and τ_2, are radically different from each other [9, 10]. The first stage (*i.e.*, transition from the arbitrary nonuniform state to the long-lived quasi-stationary state with anticorrelated distributions of T and n) takes about one unit of the dimensionless time, if the Hamiltonian (3) was written in the dimensionless form. In the physical units, this corresponds to the inverse plasma frequency:

$$\tau_1 \sim 1/\nu_{\rm pl} \, . \tag{4}$$

On the other hand, the second state (*i.e.*, a relaxation of the long-lived quasi-stationary state to the complete thermodynamic equilibrium) takes the time:

$$\tau_2 \sim N/\nu_{\rm pl} \, , \tag{5}$$

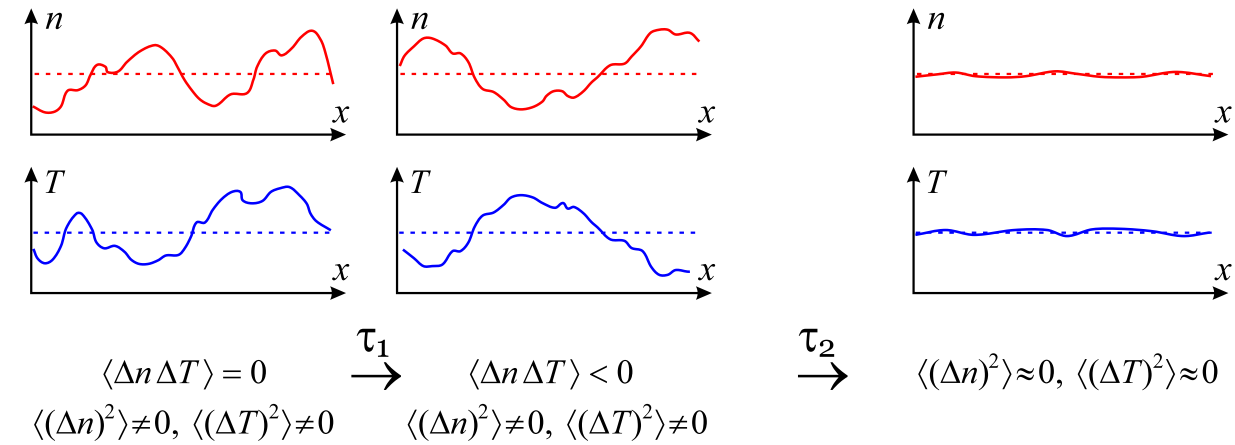

Figure 4: Sketch of two stages in the relaxation of strongly-nonequilibrium plasmas. The enhanced space between the second and third panels implies a much greater duration of the second stage of the relaxation as compared to the first stage

where N is the total number of the charged particles participating in this process. Since $N \gg 1$, then $\tau_2 \gg \tau_1$.

Just the long-lived "intermediate" state is of considerable interest for the interpretation of a few observable physical phenomena. For example, it was suggested already in the original paper [10] that the intermediate anticorrelated state of T and n might be employed for solving the well-known astrophysical problem of heating the solar corona, namely, a sharp increase of its temperature from $\sim$5000 K at the level of photosphere to $(1-2){\cdot}10^6$ K in the corona [16, 17].

Unfortunately, a more careful analysis shows that the above-mentioned mechanism cannot be related to heating of the solar corona for the following two reasons [18]. First, it assumes that the irregularities (large-scale fluctuations) of temperature and density are established in the course of self-consistent relaxation in the Coulomb system (plasma) under consideration, whereas a sharp decay of the density in the solar corona is evidently associated with gravitational stratification in the external field. Second, even if the large-scale spatial variations of temperature in the solar corona were formed by the process of relaxation predicted by the spin-type model, they would be directed randomly (upwards and downwards) in the various magnetic-flux tubes. As a result, the observed temperature would either change in the opposite directions in various regions along the solar surface or remain approximately constant (if the individual magnetic tubes were sufficiently thin and unresolved by the observations). Both these cases are evidently irrelevant to the actual situation.

On the other hand, the two-step relaxation with a long-lived intermediate anticorrelated stage, as depicted in Fig. 4, is of considerable interest just for the interpretation of the irregular structure of the solar wind. Indeed, both the above-mentioned obstacles will no longer take place here:

1. Since the solar wind propagates more or less freely, the gravitational field no longer plays a significant role in the local reference frame associated with the substance (*i.e.*, the situation becomes in a certain sense similar to the state of zero gravity inside a spaceship). Therefore, the relaxation of the temperature and density can proceed

approximately in a self-consistent manner, as required in the spin-type plasma model.

2. The distributions of the solar-wind temperature and density in the comoving reference frame really exhibit the *oppositely-directed* fluctuations of various spatial scales. (When the solar wind passes through the measuring apparatus onboard a spacecraft, these fluctuations are recorded as the changes in time.)

Therefore, Fig. 1 and the results of other papers cited in the Introduction fit well with the general predictions of the spin-type plasma models. Namely, the uncorrelated fluctuations of T and n, naturally expected in outflow of the solar wind from the corona (left panel in Fig. 4), are transformed to the long-lived anticorrelated distributions of these quantities (middle panel in the same figure).

Let us estimate the characteristic times of the respective processes by formulas (4) and (5). For the typical concentration of protons in the solar wind $n \approx 10\,\text{cm}^{-3}$, we find that the anticorrelations are formed at the time scale $\tau_1 \sim 10^{-3}\,\text{s}$, which is evidently much less than the one-minute scale of the fluctuations discussed above.

Next, to evaluate the lifetime of the anticorrelated state τ_2, we need to know the effective number of particles N in the quasi-one-dimensional "spin chain". It can be estimated as $n^{1/3}\Delta l$, where $n^{1/3}$ is the mean interparticle separation, and Δl is the characteristic spatial scale of the irregularity. At the typical speed of the solar wind 500 km/s, we get for the above-mentioned one-minute fluctuations: $\Delta l \approx 3{\cdot}10^9\,\text{cm}$. Then, formula (5) leads to $\tau_2 \sim 5{\cdot}10^6\,\text{s}$, *i.e.*, the anticorrelations can well survive even at the very long time intervals. In a sense, this estimate even looks suspiciously optimistic: it is possible that the effects of MHD turbulence destroy the anticorrelated states at the shorter time scales. This problem still needs to be studied in more detail.

Discussion and Conclusions

It is interesting to discuss in more detail the efficiency of formation of the anticorrelated distributions of temperature and density in the various phases of the solar activity cycle. Indeed, the coefficient of correlation $r = -0.13$ mentioned in the Introduction (and similar values found by other authors) was obtained for a sufficiently long period, from January 2009 to December 2017 inclusively, *i.e.*, approximately for a whole cycle. On the other hand, as follows from the detailed processing of the observational data, the coefficient r can vary substantially depending on the level of solar activity at the shorter time intervals. For example, as seen in Fig. 5, the absolute value of the correlation coefficient $|r|$ reaches the maximum values about 0.2–0.25 in the periods of low solar activity (2009–2010 and 2016–2018) and drops down to 0.05 in the period of maximum activity.

In general, such a behaviour is not surprising: the outflow of the solar wind from the corona in the periods of low activity should have a more "gentle" character, when fluctuations of temperature and density are initially random and independent from each other. Just this assumption is the basis of the model of plasma relaxation described by the spin-type Hamiltonians. On the other hand, the periods of high solar activity are characterized by a larger number of the events where the initial fluctuations of T and n are correlated positively,

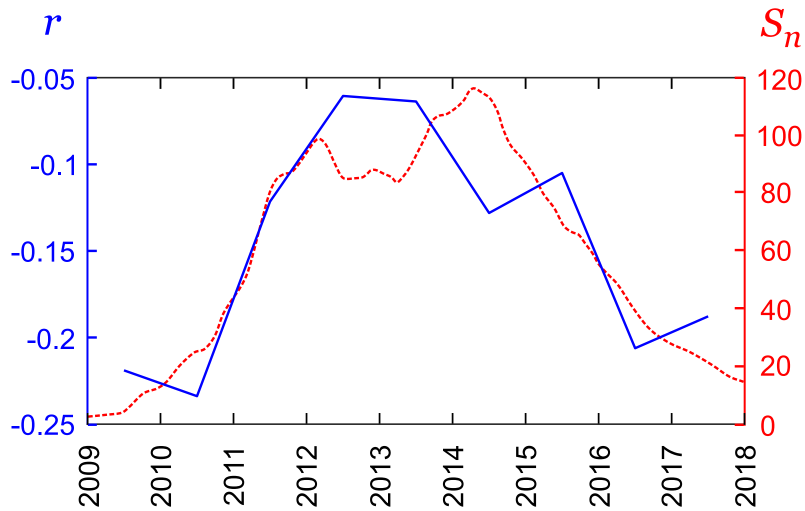

Figure 5: Variations of the correlation coefficient r (solid blue curve) depending on the level of solar activity, characterized by the mean monthly sunspot number S_n (dotted red curve) during the whole solar cycle, from 2009 to 2018.

e.g., due to the coronal mass ejections (CME) caused by a disruption of the magnetic tubes in the solar atmosphere. Indeed, as seen in Fig. 2 from paper [6], the types of the solar wind associated with ejections take up a significantly larger percentage of time in the periods of high solar activity than in the periods of low activity. The initial positive correlations between T and n in the region of solar wind formation should partially survive during its subsequent evolution in propagation to the Earth, thereby reducing the magnitude of anticorrelations in the respective periods of observations.

Finally, to avoid misunderstanding, let us emphasize that the degree of universality of the above mechanism of anticorrelations should not be exaggerated. For example, the anti-correlated distributions of temperature and density are well known in the planetary atmospheres [19, 20], but they have there a completely different nature. Roughly speaking, the increase of temperature in the rarefied outer layers of the planetary atmospheres is caused by the fact that they are located closer to the external source of irradiation—the Sun—and, therefore, absorb its radiation more efficiently. This is unrelated to the processes of relaxation in a non-equilibrium gas.

In summary, we can conclude that the method of the spin-type Hamiltonians is an efficient way to describe Langmuir turbulence in the strongly-nonequilibrium plasmas. On the one hand, this approach is based on a number of simplifying assumptions, particularly, the model of "one-component" plasma and a quite rude approximation of the Coulomb potential, as discussed above. But, on the other hand, this simplification enables us to construct a self-consistent model for relaxation of the plasma irregularities, without any additional assumptions about the characteristics of the resulting turbulence. The two-step relaxation of the strongly non-equilibrium plasmas with a long-lived intermediate stage of anticorrelated distributions of temperature and density, predicted in the framework of this model, is of considerable interest for the interpretation of the observed properties of the solar wind.

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